

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	



General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics

MFP1

Unit Further Pure 1

Tuesday 17 January 2012 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J A N 1 2 M F P 1 0 1

Answer **all** questions in the spaces provided.

1 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)

(b) Show that $\alpha^2 + \beta^2 = \frac{17}{4}$. (2 marks)

(c) Find a quadratic equation, with integer coefficients, which has roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. (5 marks)

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2 Show that only one of the following improper integrals has a finite value, and find that value:

(a) $\int_8^{\infty} x^{-\frac{2}{3}} dx;$

(b) $\int_8^{\infty} x^{-\frac{4}{3}} dx.$

(5 marks)

QUESTION
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3 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 9 = 0$; *(1 mark)*

(ii) $(x + 2)^2 + 9 = 0$. *(1 mark)*

(b) (i) Expand $(1 + x)^3$. *(1 mark)*

(ii) Express $(1 + 2i)^3$ in the form $a + bi$. *(3 marks)*

(iii) Given that $z = 1 + 2i$, find the value of
$$z^* - z^3$$
 (2 marks)

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QUESTION
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Turn over ►



4 (a) Use the formulae for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$ to show that

$$\sum_{r=1}^n r^2(4r - 3) = kn(n + 1)(2n^2 - 1)$$

where k is a constant. (5 marks)

(b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r - 3) \quad (2 \text{ marks})$$

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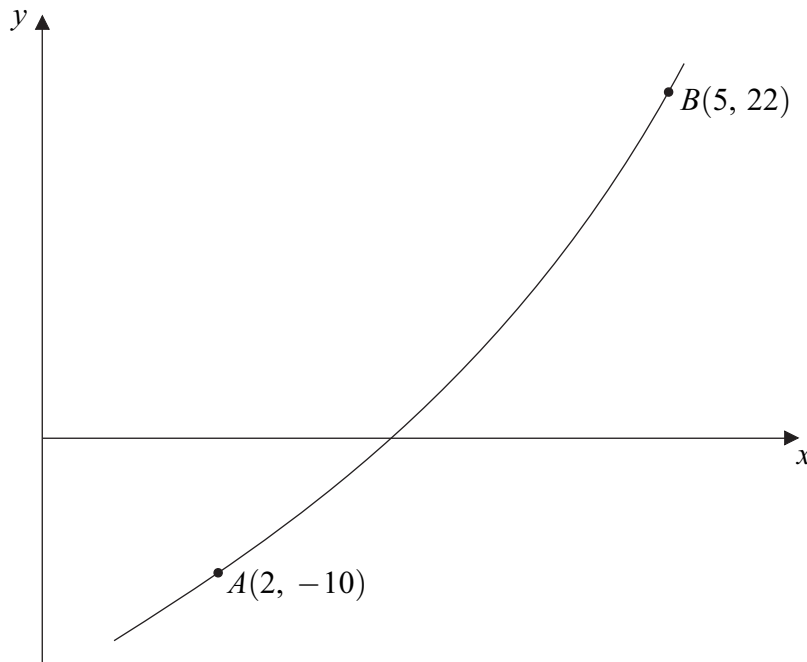
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5 The diagram below (not to scale) shows a part of a curve $y = f(x)$ which passes through the points $A(2, -10)$ and $B(5, 22)$.

- (a) (i)** On the diagram, draw a line which illustrates the method of linear interpolation for solving the equation $f(x) = 0$. The point of intersection of this line with the x -axis should be labelled P . *(1 mark)*
- (ii)** Calculate the x -coordinate of P . Give your answer to one decimal place. *(3 marks)*
- (b) (i)** On the same diagram, draw a line which illustrates the Newton–Raphson method for solving the equation $f(x) = 0$, with initial value $x_1 = 2$. The point of intersection of this line with the x -axis should be labelled Q . *(1 mark)*
- (ii)** Given that the gradient of the curve at A is 8, calculate the x -coordinate of Q . Give your answer as an exact decimal. *(2 marks)*

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6 Find the general solution of each of the following equations:

(a) $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}};$ (4 marks)

(b) $\tan^2\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{3}.$ (3 marks)

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7 A hyperbola H has equation

$$\frac{x^2}{9} - y^2 = 1$$

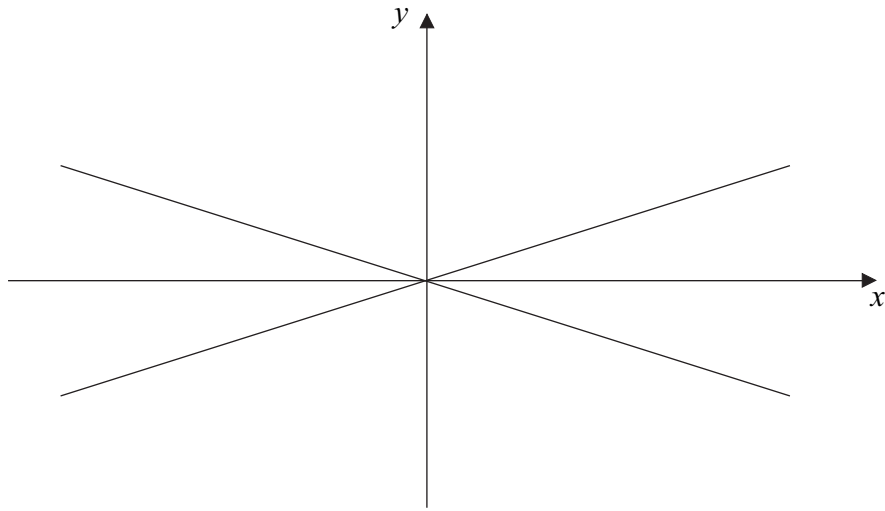
- (a) Find the equations of the asymptotes of H . (1 mark)
- (b) The asymptotes of H are shown in the diagram opposite. On the same diagram, sketch the hyperbola H . Indicate on your sketch the coordinates of the points of intersection of H with the coordinate axes. (3 marks)
- (c) The hyperbola H is now translated by the vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.
- (i) Write down the equation of the translated curve. (2 marks)
- (ii) Calculate the coordinates of the two points of intersection of the translated curve with the line $y = x$. (4 marks)
- (d) From your answers to part (c)(ii), **deduce** the coordinates of the points of intersection of the original hyperbola H with the line $y = x - 3$. (2 marks)

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(b)



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8 The diagram below shows a rectangle R_1 which has vertices $(0, 0)$, $(3, 0)$, $(3, 2)$ and $(0, 2)$.

(a) On the diagram, draw:

(i) the image R_2 of R_1 under a rotation through 90° clockwise about the origin; *(1 mark)*

(ii) the image R_3 of R_2 under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{span style="float: right;">*(3 marks)*$$

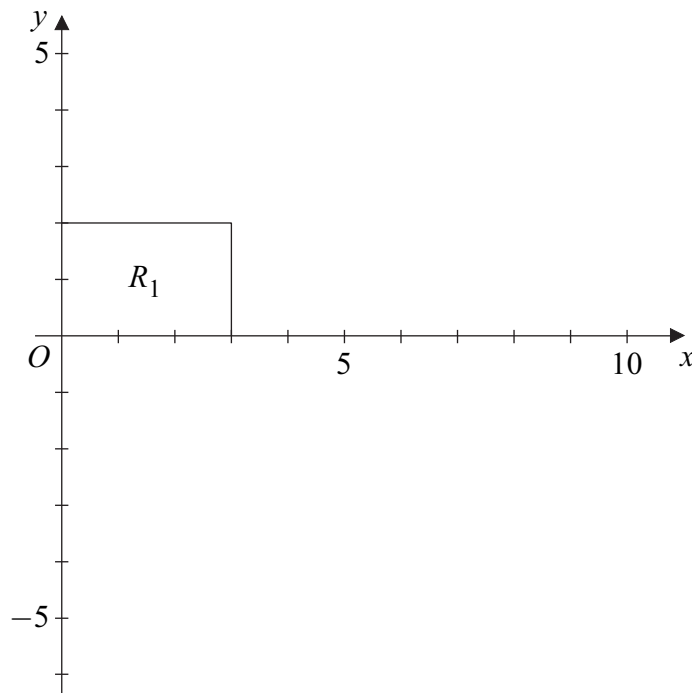
(b) Find the matrix of:

(i) the rotation which maps R_1 onto R_2 ; *(1 mark)*

(ii) the combined transformation which maps R_1 onto R_3 . *(3 marks)*

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(a)



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9 A curve has equation

$$y = \frac{x}{x-1}$$

(a) Find the equations of the asymptotes of this curve. (2 marks)

(b) Given that the line $y = -4x + c$ intersects the curve, show that the x -coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c + 3)x + c = 0 \qquad \qquad \qquad (3 \text{ marks})$$

(c) It is given that the line $y = -4x + c$ is a tangent to the curve.

(i) Find the two possible values of c .

(No credit will be given for methods involving differentiation.) (3 marks)

(ii) For each of the two values found in part **(c)(i)**, find the coordinates of the point where the line touches the curve. (4 marks)

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Dotted lines for writing.

END OF QUESTIONS



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ANSWER IN THE SPACES PROVIDED**

